Introduction to Increased Limits Ratemaking

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Increased Limits Ratemaking is the process of developing charges for expected losses at higher limits of liability.

Increased Limits Ratemaking is the process of developing charges for expected losses at higher limits of liability.

Expressed as a factor --- an Increased Limit Factor --- to be applied to basic limits loss costs

Calculation Method

$$\frac{\text{Expected Costs at the desired policy limit}}{\text{Expected Costs at the Basic Limit}}$$

KEY ASSUMPTION:

Claim Frequency is <u>independent</u> of

Claim Severity

This allows for ILFs to be developed by an examination of the relative severities ONLY

$$ILF_k = \frac{E(Frequency) \times E(Severity_k)}{E(Frequency) \times E(Severity_B)}$$

$$= \frac{E(Severity_k)}{E(Severity_B)}$$

Limited Average Severity (LAS)

- Defined as the average size of loss, where all losses are limited to a particular value.
- Thus, the ILF can be defined as the ratio of two limited average severities.
- ILF (k) = LAS (k) ÷ LAS (B)

Example

Losses	@100,000 Limit	@1 Mill Limit
50,000		
75,000		
150,000		
250,000		
1,250,000		

Example (cont'd)

Losses	@100,000 Limit	@1 Mill Limit
50,000	50,000	
75,000	75,000	
150,000	100,000	
250,000	100,000	
1,250,000	100,000	

Example (cont'd)

Losses	@100,000 Limit	@1 Mill Limit
50,000	50,000	50,000
75,000	75,000	75,000
150,000	100,000	150,000
250,000	100,000	250,000
1,250,000	100,000	1,000,000

Example – Calculation of ILF

Total Losses	$1,775,000
Limited to $100,000 (Basic Limit)	$425,000
Limited to $1,000,000	$1,525,000
Increased Limits Factor (ILF)	3.588

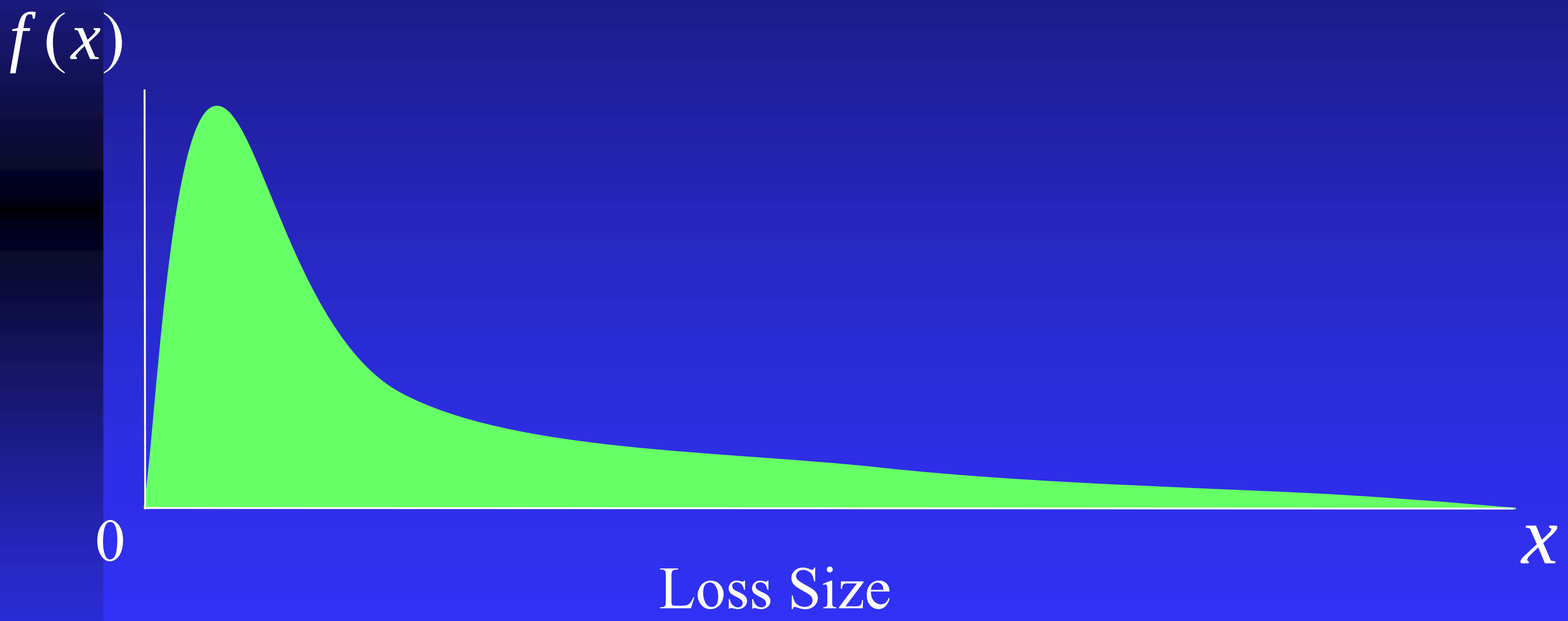
Insurance Loss Distributions

- Loss Severity Distributions are Skewed
- Many Small Losses/Fewer Larger Losses
- Yet Larger Losses, though fewer in number, are a significant amount of total dollars of loss.

Basic Limits vs. Increased Limits

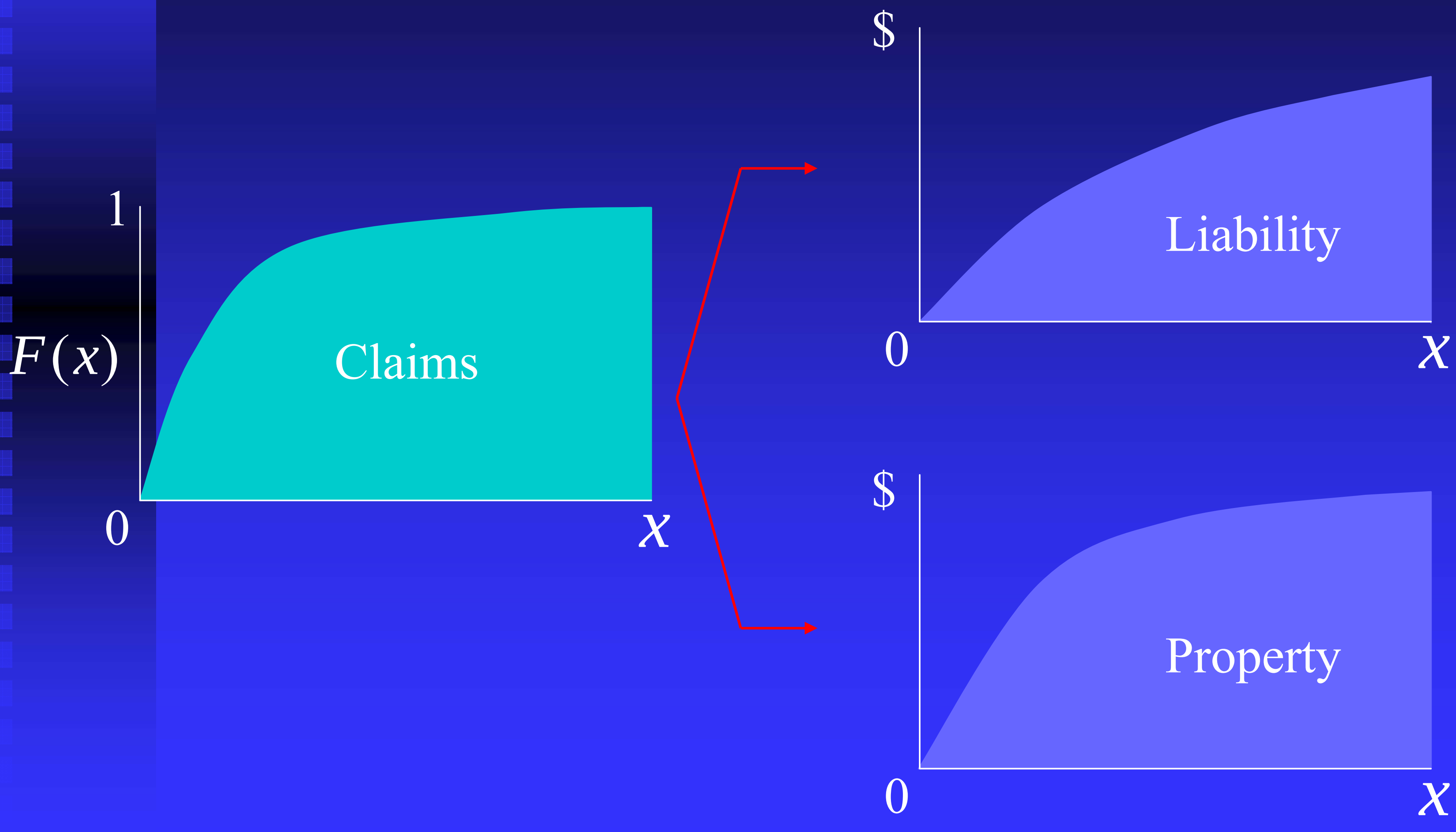
- Use large volume of losses capped at basic limit for detailed, experience-based analysis.
- Use a broader experience base to develop ILFs to price higher limits

Loss Distribution - PDF



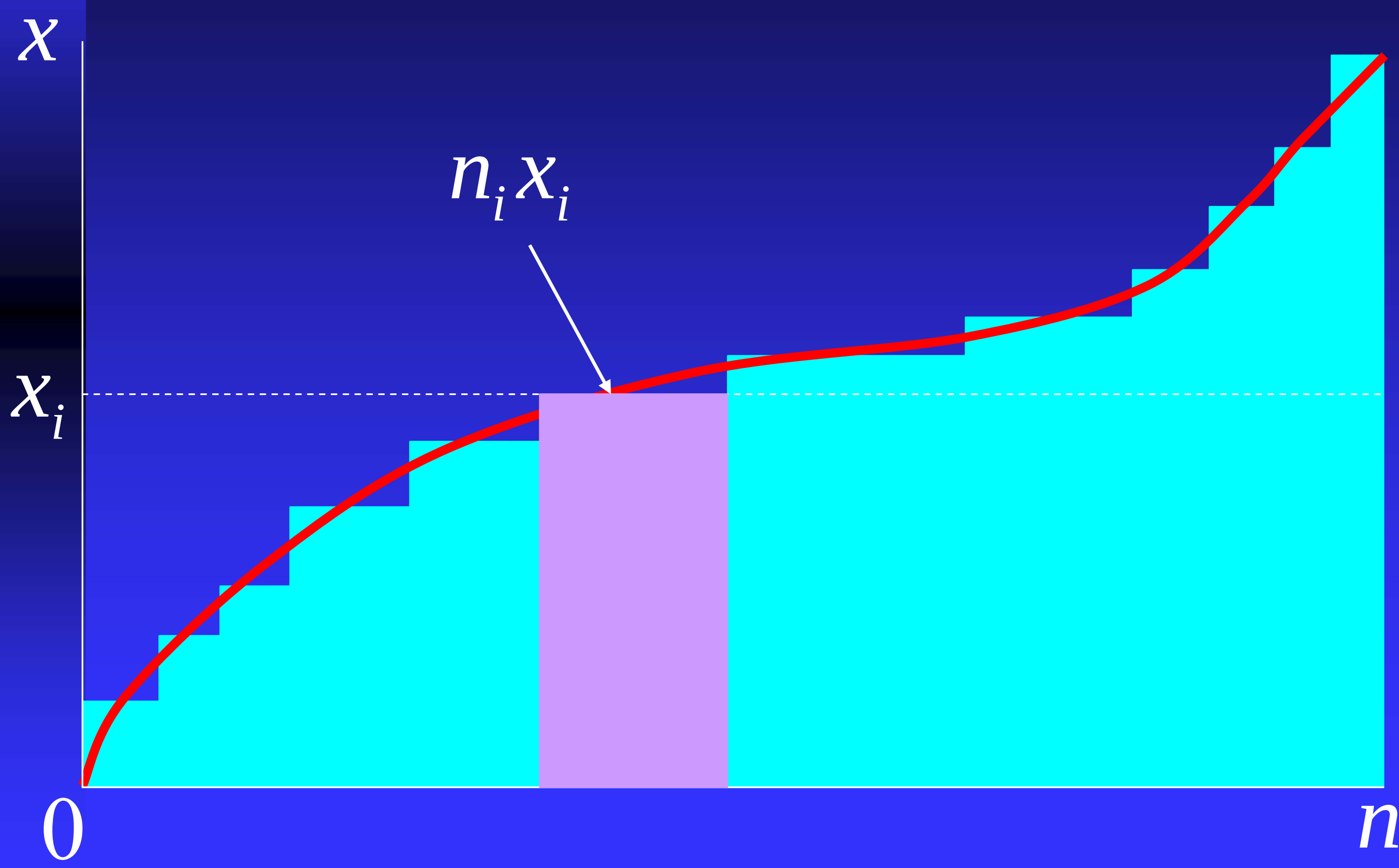
Loss Distribution - CDF

Claims vs. Cumulative Paid $



A novel approach to understanding Increased Limits Factors was presented by Lee in the paper --- “The Mathematics of Excess of Loss Coverages and Retrospective Rating - A Graphical Approach”

Lee (Figure 1)



Limited Average Severity

$$\int_0^k x\,dF(x) + k[1 - F(k)]$$

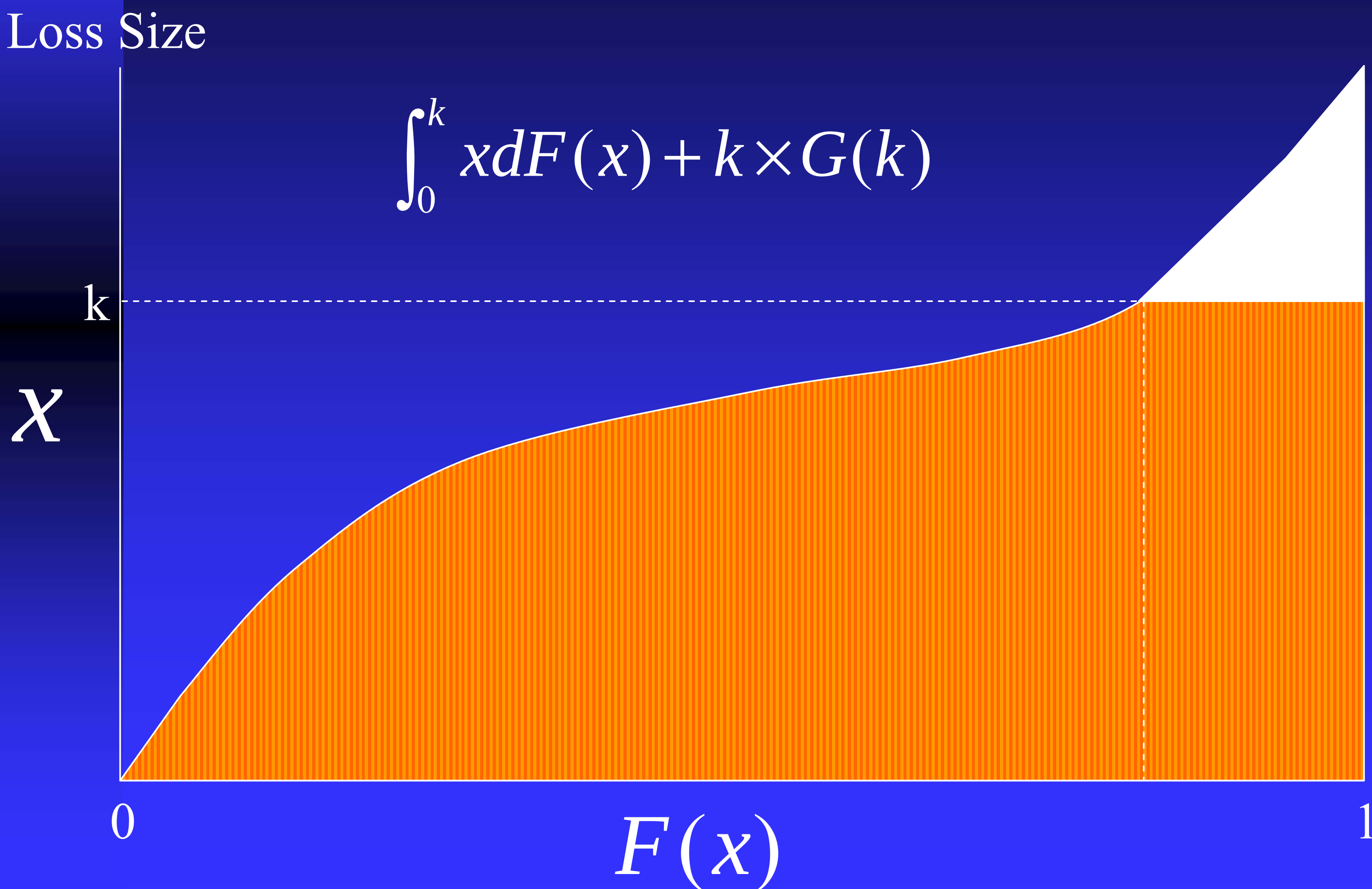
Size method; ‘vertical’

$$\int_0^k [1 - F(x)]\,dx$$

Layer method; ‘horizontal’

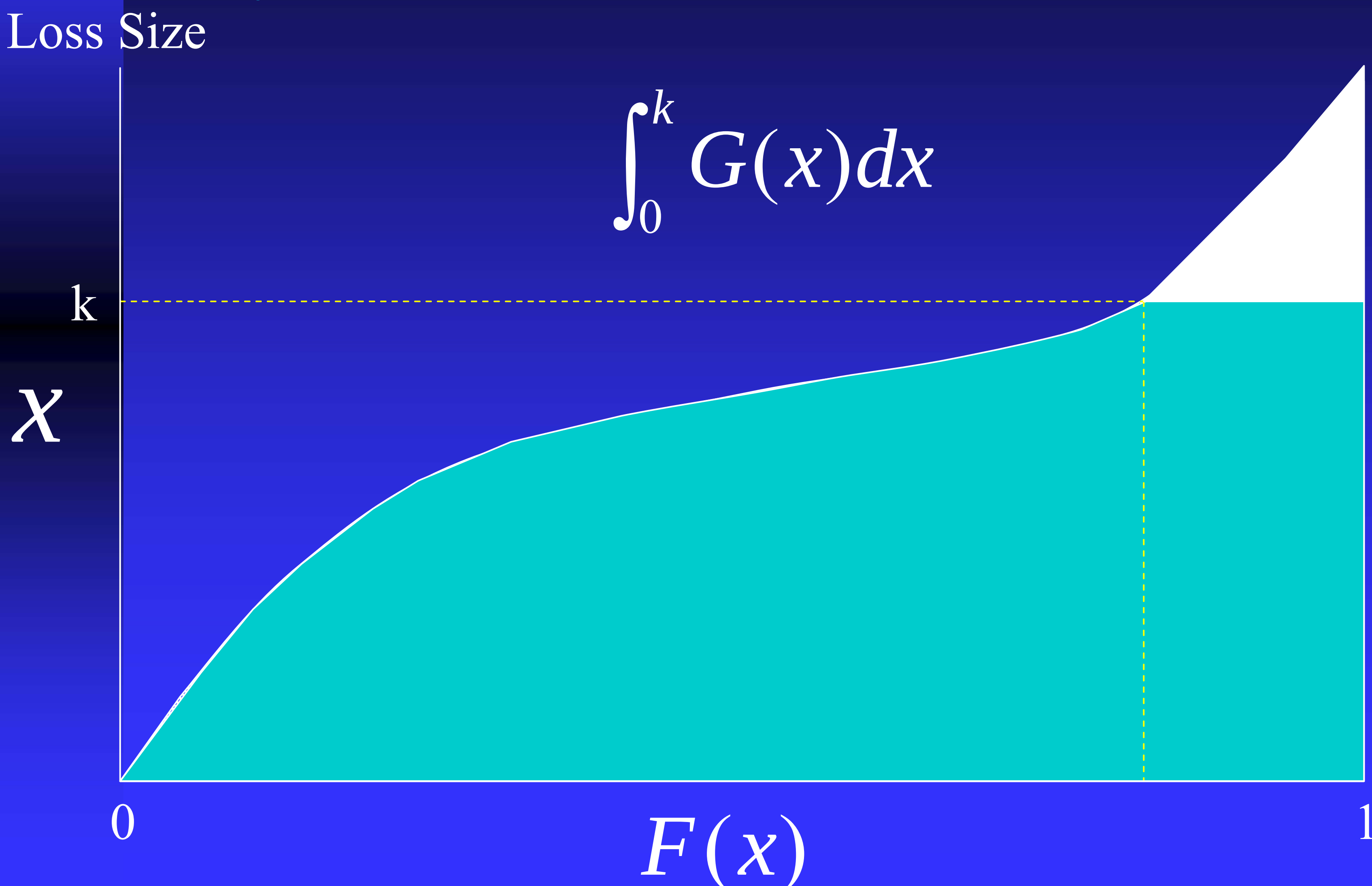
$$*\,G(x) = 1 - F(x)$$

Size Method



$$*G(x) = 1 - F(x)$$

Layer Method



Empirical Data - ILFs

Lower	Upper	Losses	Occs.	LAS
1	100,000	25,000,000	1,000	
100,001	250,000	75,000,000	500	
250,001	500,000	60,000,000	200	
500,001	1 Million	30,000,000	50	
1 Million	-	15,000,000	10	-

Empirical Data - ILFs

LAS @ 100,000

$$(25{,}000{,}000 + 760 \times 100{,}000) \div 1760$$

$$= 57{,}386$$

LAS @ 1,000,000

$$(190{,}000{,}000 + 10 \times 1{,}000{,}000) \div 1760$$

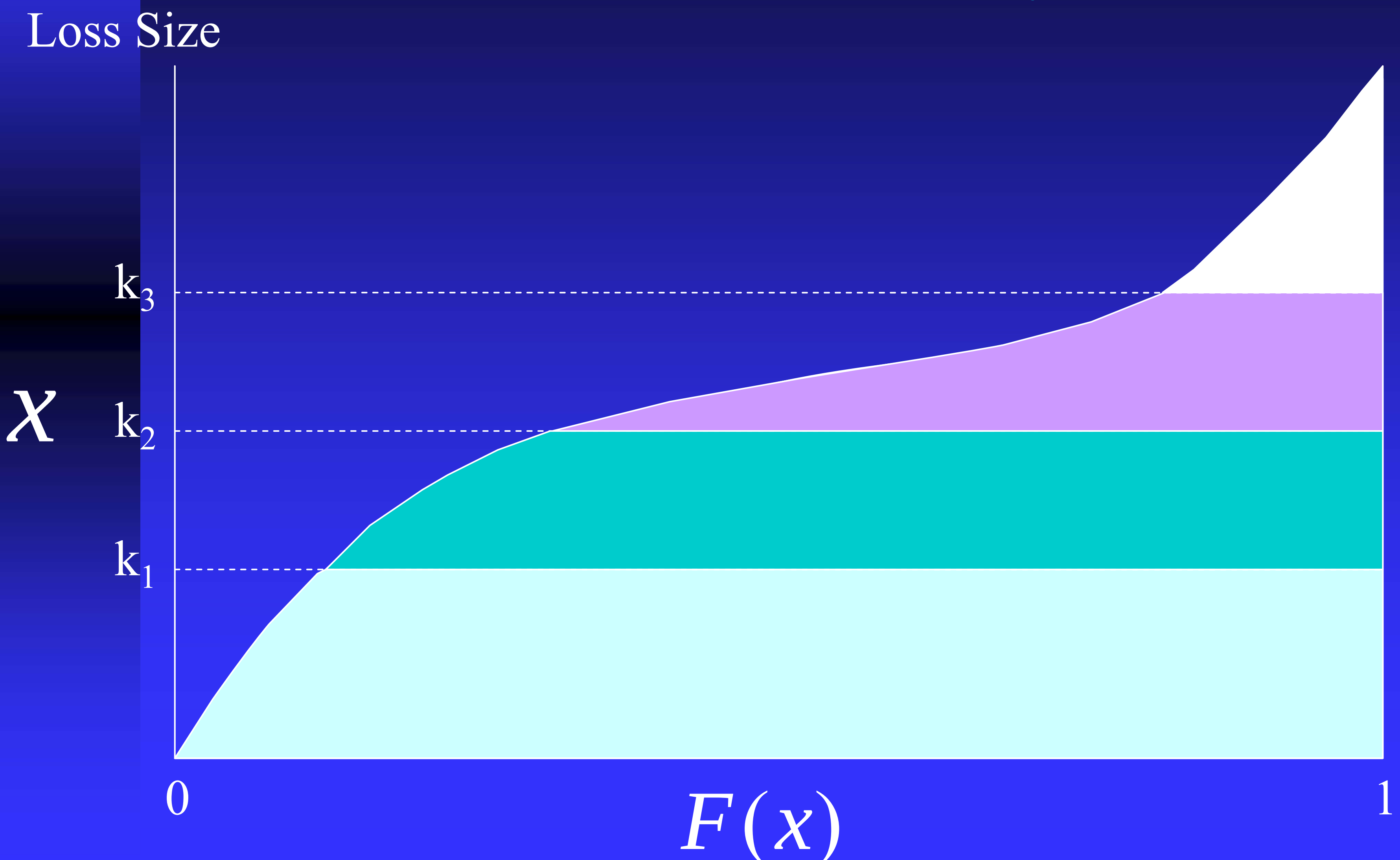
$$= 113{,}636$$

Empirical ILF = 1.98

“Consistency” of ILFs

- As Policy Limit Increases
  - ILFs should increase
  - But at a decreasing rate
- Expected Costs per unit of coverage should not increase in successively higher layers.

Illustration: Consistency



“Consistency” of ILFs - Example

Limit	ILF	Diff. Lim.	Diff. ILF	Marginal
100,000	1.00	-	-	-
250,000	1.40			
500,000	1.80			
1 Million	2.75			
2 Million	4.30			
5 Million	5.50			

“Consistency” of ILFs - Example

Limit	ILF	Diff. Lim.	Diff. ILF	Marginal
100,000	1.00	-	-	-
250,000	1.40	150	0.40	
500,000	1.80	250	0.40	
1 Million	2.75	500	0.95	
2 Million	4.30	1,000	1.55	
5 Million	5.50	3,000	1.20	

"Consistency" of ILFs - Example

Limit	ILF	Diff. Lim.	Diff. ILF	Marginal
100,000	1.00	-	-	-
250,000	1.40	150	0.40	.0027
500,000	1.80	250	0.40	.0016
1 Million	2.75	500	0.95	.0019
2 Million	4.30	1,000	1.55	.00155
5 Million	5.50	3,000	1.20	.0004

“Consistency” of ILFs - Example

Limit	ILF	Diff. Lim.	Diff. ILF	Marginal
100,000	1.00	-	-	-
250,000	1.40	150	0.40	.0027
500,000	1.80	250	0.40	.0016
1 Million	2.75	500	0.95	.0019*
2 Million	4.30	1,000	1.55	.00155
5 Million	5.50	3,000	1.20	.0004

Components of ILFs

- Expected Loss
- Allocated Loss Adjustment Expense (ALAE)
- Unallocated Loss Adjustment Expense (ULAE)
- Parameter Risk Load
- Process Risk Load

ALAE

- Claim Settlement Expense that can be assigned to a given claim --- primarily Defense Costs
- Loaded into Basic Limit
- Consistent with Duty to Defend Insured
- Consistent Provision in All Limits

Unallocated LAE – (ULAE)

- Average Claims Processing Overhead Costs
  - e.g. Salaries of Claims Adjusters
- Percentage Loading into ILFs for All Limits
  - Average ULAE as a percentage of Losses plus ALAE
  - Loading Based on Financial Data

Process Risk Load

- Process Risk --- the inherent variability of the insurance process, reflected in the difference between actual losses and expected losses.
- Charge varies by limit

Parameter Risk Load

- Parameter Risk --- the inherent variability of the estimation process, reflected in the difference between theoretical (true but unknown) expected losses and the estimated expected losses.
- Charge varies by limit

Increased Limits Factors (ILFs)

ILF @ Policy Limit (k) is equal to:

$$\frac{LAS(k) + ALAE(k) + ULAE(k) + RL(k)}{LAS(B) + ALAE(B) + ULAE(B) + RL(B)}$$

Components of ILFs

Limit	LAS	ALAE	ULAE	PrRL	PaRL	ILF
100	7,494	678	613	76	79	1.00
250	8,956	678	723	193	94	1.19
500	10,265	678	821	419	108	1.37
1,000	11,392	678	905	803	123	1.55
2,000	12,308	678	974	1,432	135	1.74

Deductibles

- Types of Deductibles
- Loss Elimination Ratio
- Expense Considerations

Types of Deductibles

- Reduction of Damages
  - Insurer is responsible for losses in excess of the deductible, up to the point where an insurer pays an amount equal to the policy limit
  - An insurer may pay for losses in layers above the policy limit (But, total amount paid will not exceed the limit)
- Impairment of Limits
  - The maximum amount paid is the policy limit minus the deductible

Deductibles (example 1)

Example 1:

Policy Limit:	$100,000
Deductible:	$25,000
Occurrence of Loss:	$100,000

Reduction of Damages

Loss - Deductible

=100,000 - 25,000=75,000

(Payment up to Policy Limit)

Payment is $75,000

Reduction due to Deductible is $25,000

Impairment of Limits

Loss does not exceed Policy Limit, so:

Loss - Deductible

=100,000 - 25,000=75,000

Payment is $75,000

Reduction due to Deductible is $25,000

Deductibles (example 2)

Example 2:

Policy Limit: $100,000

Deductible: $25,000

Occurrence of Loss: $125,000

Reduction of Damages

Loss - Deductible

=125,000 - 25,000=100,000

(Payment up to Policy Limit)

Payment is $100,000

Reduction due to Deductible is $0

Impairment of Limits

Loss exceeds Policy Limit, so:

Policy Limit - Deductible

=100,000 - 25,000=75,000

Payment is $75,000

Reduction due to Deductible is $25,000

Liability Deductibles

- Reduction of Damages Basis
- Apply to third party insurance
- Insurer handles all claims
    - Loss Savings
    - No Loss Adjustment Expense Savings
- Deductible Reimbursement
    - Risk of Non-Reimbursement
- Discount Factor

Deductible Discount Factor

- Two Components
  - Loss Elimination Ratio (LER)
  - Combined Effect of Variable & Fixed Expenses
    - This is referred to as the Fixed Expense Adjustment Factor (FEAF)

Loss Elimination Ratio

- Net Indemnity Costs Saved – divided by Total Basic Limit/Full Coverage Indemnity & LAE Costs
- Denominator is Expected Basic Limit Loss Costs

Loss Elimination Ratio (cont'd)

- Deductible (i)
- Policy Limit (j)
- Consider [LAS(i+j) - LAS(i)] ÷ LAS(j)
- This represents costs under deductible as a fraction of costs without a deductible.
- One minus this quantity is the (indemnity) LER
- Equal to

  [LAS(j) - LAS(i+j) + LAS(i)] ÷ LAS(j)

Loss Elimination Ratio (cont'd)

- LAS(j) – LAS(i+j) + LAS(i) represents the Gross Savings from the deductible.
- Need to multiply by the Business Failure Rate
  - Accounts for risk that insurer will not be reimbursed
- Net Indemnity Savings

  $= \text{Gross Savings} \times (1 - \text{BFR})$

Fixed Expense Adjustment Factor

- Deductible Savings do not yield Fixed Expense Savings
- Variable Expense Ratio (VER)
  - Percentage of Premium
  - So: Total Costs Saved from deductible equals Net Indemnity Savings divided by (1-VER)

FEAF (cont'd)

- Now: Basic Limit Premium equals Basic Limit Loss Costs divided by the Expected Loss Ratio (ELR)
- We are looking for:

Total Costs Saved ÷ Basic Limit Premium

FEAF (cont'd)

$$\frac{\text{Total Costs Saved} \div \text{Basic Limit Premium}}{} $$

Total Costs Saved ÷ Basic Limit Premium

Is Equivalent to:

$$\frac{\text{Net Indemnity Savings} \div (1\text{-VER})}{\text{Basic Limit Loss Costs} \div \text{ELR}}$$

Which equals: LER × [ELR ÷ (1-VER)]

So: FEAF = ELR ÷ (1 - VER)

Deductibles – Summary

$$\text{Deductible Discount Factor} = \text{Fixed Expense Adjustment Factor (FEAF)} \times \text{Loss Elimination Ratio (LER)}$$

$$\text{FEAF} = \frac{\text{Expected Loss Ratio}}{1 - \text{Variable Expense Ratio}}$$

$$\text{LER} = \frac{\text{Expected Net Indemnity Savings}}{\text{Total Expected B.L. Indemnity} + \text{ALAE} + \text{ULAE}}$$

A Numerical Example

Expected Losses	65	Premium	100
Fixed Expenses	5	ELR	.65
VER	.30	FEAF	.929
Net LER	.10		
Deductible Discount Factor = .0929			
New Premium = 90.71			

Numerical Example (cont'd)

New Net Expected Losses = (1 - .10) × 65

= 58.5

Add Fixed Expenses ➡ 58.5 + 5

= 63.5

Divide by (1 - VER) ➡ 63.5 ÷ .70

= 90.71

Which agrees with our previous calculation

$$*G(x) = 1 - F(x)$$

Limited Average Severity - Layer

$$\int_{k_1}^{k_2} x\,dF(x) + k_2 \times G(k_2) - k_1 \times G(k_1)$$

Size method; ‘vertical’

$$\int_{k_1}^{k_2} G(x)\,dx$$

Layer method; ‘horizontal’

$$*G(x) = 1 - F(x)$$

Size Method & LAS

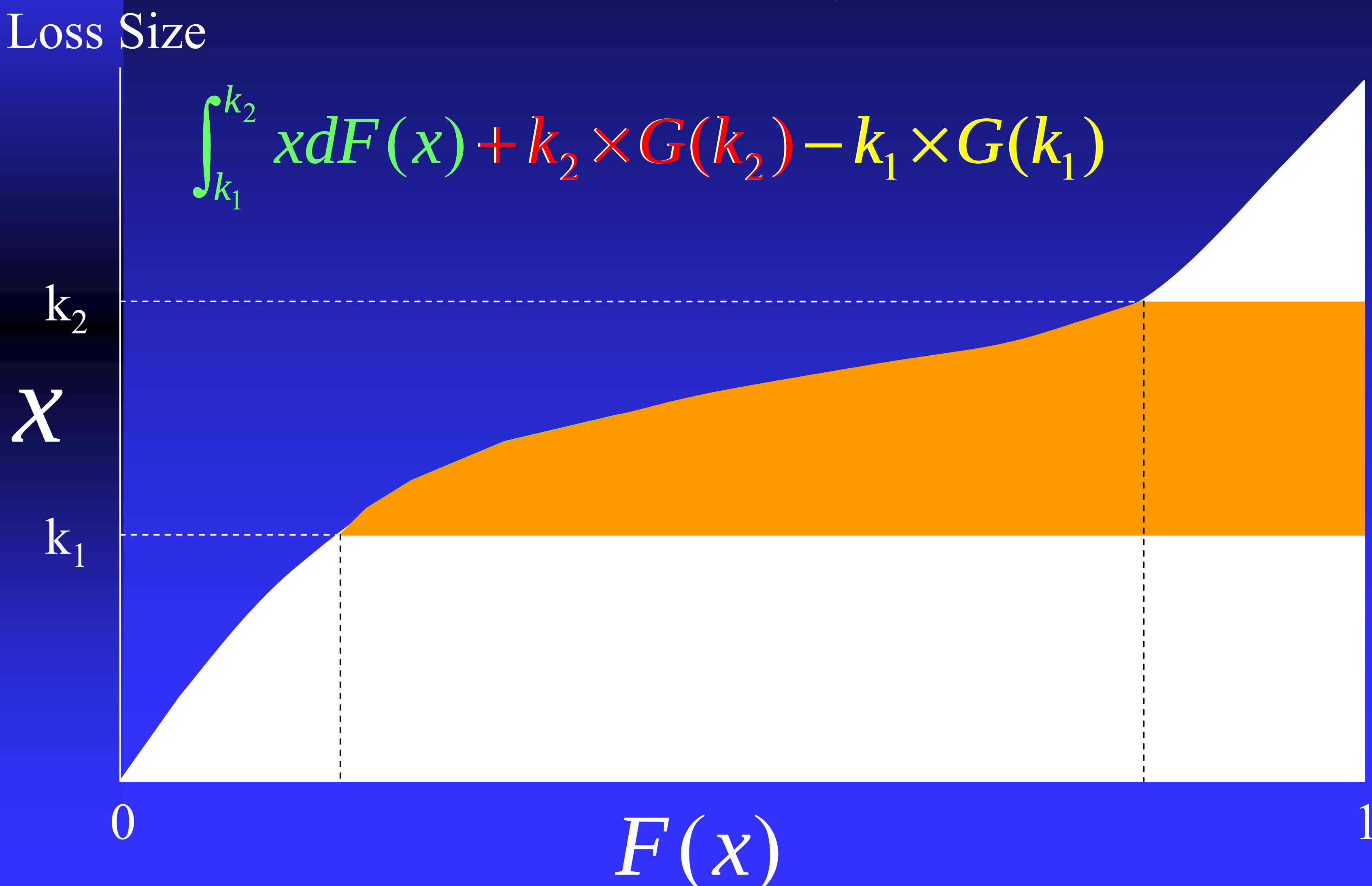
$$\int_{k_1}^{k_2} x\,dF(x) + k_2 \times G(k_2) - k_1 \times G(k_1)$$

is equal to

$$\left[\int_{0}^{k_2} x\,dF(x) + k_2 \times G(k_2)\right] - \left[\int_{0}^{k_1} x\,dF(x) + k_1 \times G(k_1)\right]$$

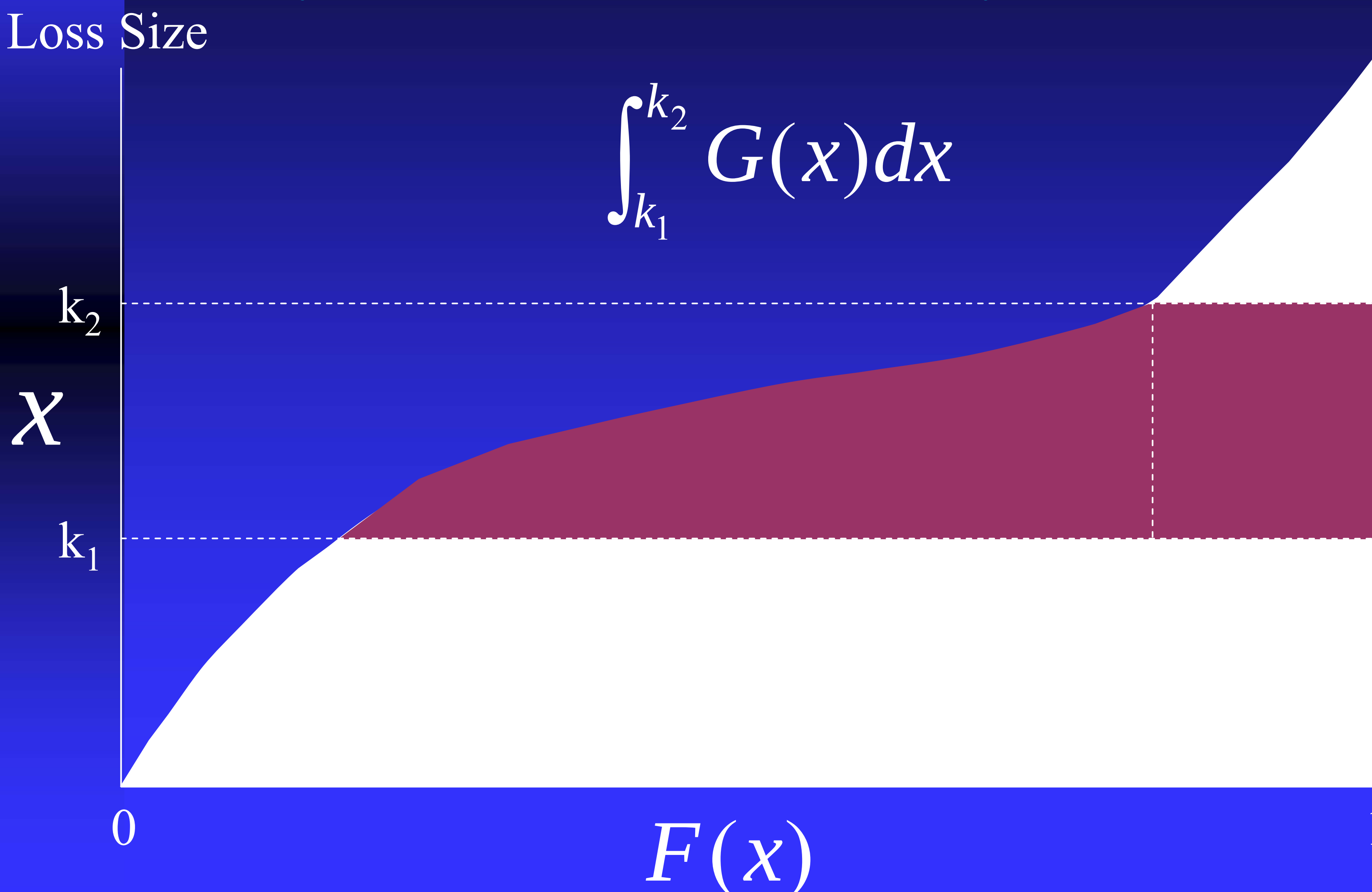
$$*G(x)=1-F(x)$$

Size Method – Layer of Loss



$$*G(x) = 1 - F(x)$$

"Layer Method" – Layer of Loss



Layers of Loss

- Expected Loss
- ALAE
- ULAE
- Risk Load

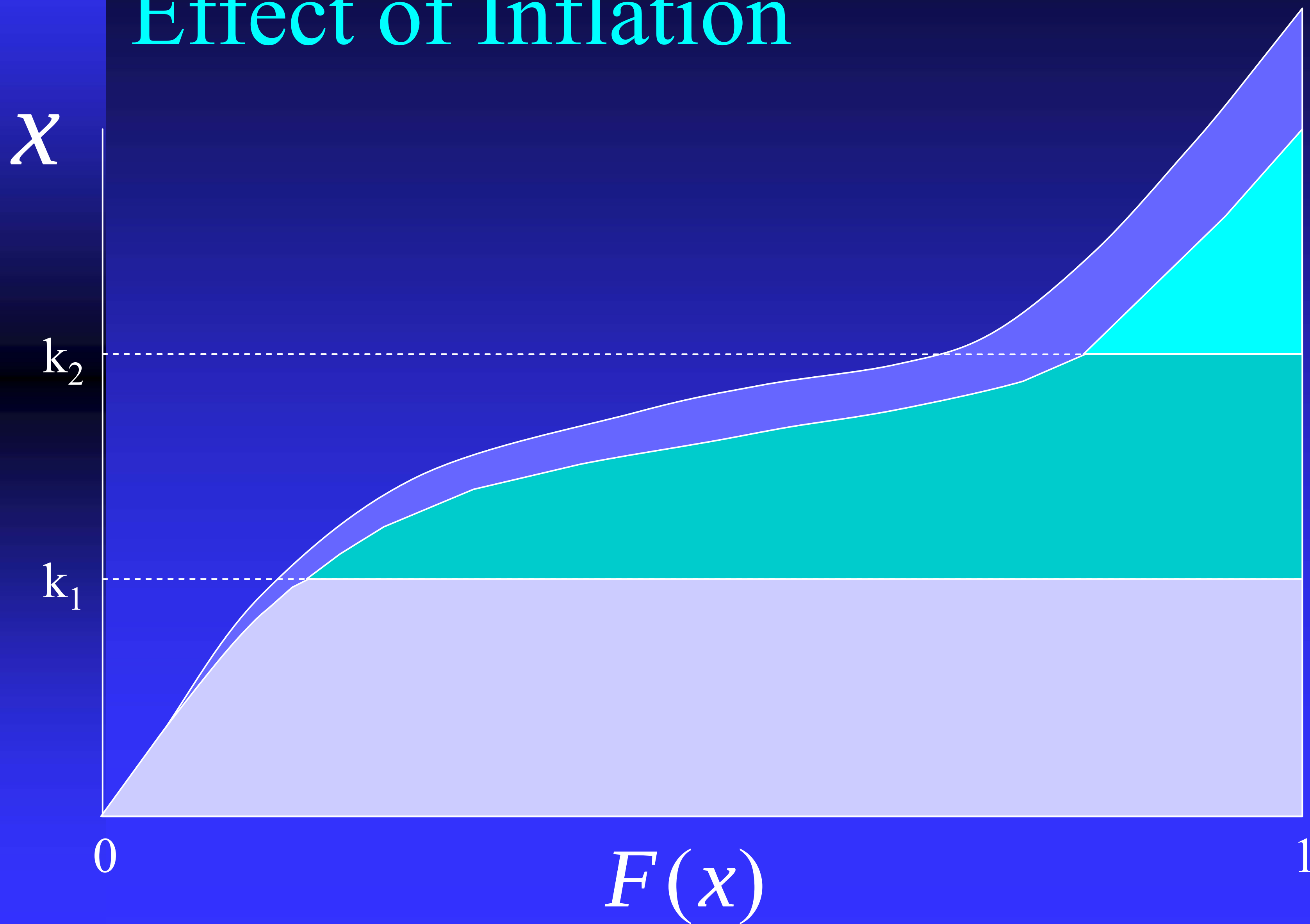
Inflation – Leveraged Effect

Generally, trends for higher limits will be higher than basic limit trends.

Also, Excess Layer trends will generally exceed total limits trends.

Requires steadily increasing trend.

Effect of Inflation



Example: Effect of +10% Trend @ $100,000 Limit

Loss Amount ($)	@ $100,000 Limit	
	Pre-Trend ($)	Post-Trend ($)
50,000	50,000	55,000
250,000	100,000	100,000
490,000	100,000	100,000
750,000	100,000	100,000
925,000	100,000	100,000
1,825,000	100,000	100,000
Total	550,000	555,000
Realized Trend	+0.9%	

Example: Effect of +10% Trend @ $500,000 Limit

Loss Amount ($)	@ $500,000 Limit	
	Pre-Trend ($)	Post-Trend ($)
50,000	50,000	55,000
250,000	250,000	275,000
490,000	490,000	500,000
750,000	500,000	500,000
925,000	500,000	500,000
1,825,000	500,000	500,000
Total	2,290,000	2,330,000
Realized Trend	+1.7%	

Example: Effect of +10% Trend @ $1,000,000 Limit

Loss Amount ($)	@ $1,000,000 Limit	
	Pre-Trend ($)	Post-Trend ($)
50,000	50,000	55,000
250,000	250,000	275,000
490,000	490,000	539,000
750,000	750,000	825,000
925,000	925,000	1,000,000
1,825,000	1,000,000	1,000,000
Total	3,465,000	3,694,000
Realized Trend	+6.6%	

Example: Effect of +10% Trend
$250,000 xs $250,000

Loss Amount ($)	$250,000 excess of $250,000 layer	
	Pre-Trend ($)	Post-Trend ($)
50,000	-	-
250,000	-	25,000
490,000	240,000	250,000
750,000	250,000	250,000
925,000	250,000	250,000
1,825,000	250,000	250,000
Total	990,000	1,025,000
Realized Trend	+3.5%	

Example: Effect of +10% Trend $500,000 xs $500,000

Loss Amount ($)	$500,000 excess of $500,000 layer	
	Pre-Trend ($)	Post-Trend ($)
50,000	-	-
250,000	-	-
490,000	-	39,000
750,000	250,000	325,000
925,000	425,000	500,000
1,825,000	500,000	500,000
Total	1,175,000	1,364,000
Realized Trend	+16.1%	

Example: Effect of +10% Trend $1,000,000 xs $1,000,000

Loss Amount ($)	$1,000,000 excess of $1,000,000 layer	
	Pre-Trend ($)	Post-Trend ($)
50,000	-	-
250,000	-	-
490,000	-	-
750,000	-	-
925,000	-	17,500
1,825,000	825,000	1,000,000
Total	825,000	1,017,500
Realized Trend	+23.3%	

Commercial Automobile Policy Limit Distribution

- ISO Database Composition (Approx.):
  - 70% - 95% at $1 Million Limit
  - 1% - 15% at $500,000 Limit
  - 1% - 15% at $2 Million Limit
- Varies by Table and State Group

Commercial Automobile
Bodily Injury

- Data Through 6/30/2005
- Paid Loss Data --- $100,000 Limit
  - 12-point: + 4.4%
  - 24-point: + 5.8%

Commercial Automobile
Bodily Injury

- Data Through 6/30/2005
- Paid Loss Data --- $1 Million Limit
  - 12-point: + 6.6%
  - 24-point: + 9.3%

Commercial Automobile
Bodily Injury

- Data Through 6/30/2005
- Paid Loss Data --- Total Limits
  - 12-point: + 7.2%
  - 24-point: + 10.3%

Mixed Exponential Methodology

Issues with Constructing ILF Tables

- Policy Limit Censorship
- Excess and Deductible Data
- Data is from several accident years
  - Trend
  - Loss Development
- Data is Sparse at Higher Limits

Use of Fitted Distributions

- May address these concerns
- Enables calculation of ILFs for all possible limits
- Smoothes the empirical data
- Examples:
  - Truncated Pareto
  - Mixed Exponential

Mixed Exponential Distribution

- Trend
- Construction of Empirical Survival Distributions
- Payment Lag Process
- Tail of the Distribution
- Fitting a Mixed Exponential Distribution
- Final Limited Average Severities

Trend

- Multiple Accident Years are Used
- Each Occurrence is trended from the average date of its accident year to one year beyond the assumed effective date.

Empirical Survival Distributions

- Paid Settled Occurrences are Organized by Accident Year and Payment Lag.
- After trending, a survival distribution is constructed for each payment lag, using discrete loss size layers.
- Conditional Survival Probabilities (CSPs) are calculated for each layer.
- Successive CSPs are multiplied to create ground-up survival distribution.

Conditional Survival Probabilities

- The probability that an occurrence exceeds the upper bound of a discrete layer, given that it exceeds the lower bound of the layer is a CSP.
- Attachment Point must be less than or equal to lower bound.
- Policy Limit + Attachment Point must be greater than or equal to upper bound.

Empirical Survival Distributions

- Successive CSPs are multiplied to create ground-up survival distribution.
- Done separately for each payment lag.
- Uses 52 discrete size layers.
- Allows for easy inclusion of excess and deductible loss occurrences.

Payment Lag Process

- Payment Lag =

  (Payment Year – Accident Year) + 1
- Loss Size tends to increase at higher lags
- Payment Lag Distribution is Constructed
- Used to Combine By-Lag Empirical Loss Distributions to generate an overall Distribution
- Implicitly Accounts for Loss Development

Payment Lag Process

- Payment Lag Distribution uses three parameters R1, R2, R3

$$R1 = \frac{\text{Expected \% of Occ. Paid in lag 2}}{\text{Expected \% of Occ. Paid in lag 1}}$$

$$R2 = \frac{\text{Expected \% of Occ. Paid in lag 3}}{\text{Expected \% of Occ. Paid in lag 2}}$$

$$R3 = \frac{\text{Expected \% of Occ. Paid in lag (n+1)}}{\text{Expected \% of Occ. Paid in lag (n)}}$$

(Note that lags 5 and higher are combined – C. Auto)

Lag Weights

- Lag 1 wt. = 1 ÷ k
- Lag 2 wt. = R1 ÷ k
- Lag 3 wt. = R1 × R2 ÷ k
- Lag 4 wt. = R1 × R2 × R3 ÷ k
- Lag 5 wt. = R1 × R2 × [$R3^2$ ÷ (1 - R3)] ÷ k
- Where k = 1 + R1 + [R1 × R2] ÷ [1 - R3]

Lag Weights

- Represent % of ground-up occurrences in each lag
- Umbrella/Excess policies not included
- R1, R2, R3 estimated via maximum likelihood.

Tail of the Distribution

- Data is sparse at high loss sizes
- An appropriate curve is selected to model the tail (e.g. a Truncated Pareto).
- Fit to model the behavior of the data in the highest credible intervals – then extrapolate.
- Smoothes the tail of the distribution.
- A Mixed Exponential is now fit to the resulting Survival Distribution Function

Simple Exponential

- Mean parameter: μ
- Policy Limit: PL

$$SDF(x) = e^{-x/\mu} = 1 - CDF(x)$$

$$LAS(PL) = \mu[1 - e^{-PL/\mu}]$$

Mixed Exponential

- Weighted Average of Exponentials
- Each Exponential has Two Parameters mean (μ_i) and weight (w_i)
- Weights sum to unity

*PL: Policy Limit

$$SDF(x) = \sum_i [w_i e^{-x/\mu_i}]$$

$$LAS(PL) = \sum_i w_i \mu_i [1 - e^{-PL/\mu_i}]$$

Mixed Exponential

- Number of individual exponentials can vary
- Generally between four and six
- Highest mean limited to 10,000,000

Sample of Actual Fitted Distribution

Mean	Weight
4,100	0.802804
32,363	0.168591
367,341	0.023622
1,835,193	0.004412
10,000,000	0.000571

Calculation of LAS

$$LAS(PL) = \sum_i w_i \mu_i [1 - e^{-PL/\mu_i}]$$

*PL: Policy Limit

$$LAS(100,000) = 11,054$$

$$LAS(1,000,000) = 20,800$$

$$ILF = \frac{LAS(1,000,000)}{LAS(100,000)} = \frac{20,800}{11,054} = 1.88$$

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